Spatio-Temporal Analysis of Gasoline Shortage in Tohoku Region after the Great East Japan Earthquake

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Outline

1. Introduction
2. Fuel Distribution Model
3. Parameter Estimation
4. Spatial Distribution of D-S Gap
5. Conclusion
6. Appendix
Although oil terminals along the Sea of Japan were restored soon after the earthquake, those on the Pacific Ocean coast remained defective.

Micro-scopic (e.g. municipality-wise) analyses of the timing of resolution of the supply shortage are required.
Purpose

Develop a model for estimating the **spatio-temporal distribution** of demand-supply (D-S) gap, which consists of:

1. **Inter**-temporal dynamics of fuel demand with **disappearance** due to a huge backlog
2. **Intra**-temporal fuel assignments taking into account the **fairness** among municipalities.

Estimate model parameters and obtain spatio-temporal distributions of D-S gap.
Framework

- $T = \{1, 2, \cdots , T\}$: set of time periods with an interval $\Delta t$;
- $O = \{1, 2, \cdots , O\}$: set of oil terminals (fuel suppliers);
- $D = \{1, 2, \cdots , D\}$: set of municipalities (fuel consumers).
- Time sequence

\[
\begin{align*}
t &\quad \text{fuel assignment at } t \\
(1 - \beta \Delta t)X_j(t - 1) &\quad \text{carry over from } t - 1 \\
r_j(t)\Delta t &\quad \text{new demand} \\
\sum_{i \in O} x_{ij}(t) &\quad \text{resolved demand} \\
\beta X(t)\Delta t &\quad \text{disappear} \\
(1 - \beta \Delta t)X_j(t) &\quad \text{carry over to } t + 1
\end{align*}
\]
Fuel Demand Dynamics (inter-temporal model)

The fuel demand queue at the end of period $t$ follows the dynamics:

$$X_j(t) = (1 - \beta \Delta t)X_j(t - 1) + r_j(t)\Delta t - \sum_{i \in O} x_{i,j}(t)\Delta t$$

- $X_j(t)$: Unresolved demand stock (i.e. “queue”) at the end of period $t$ of municipality $j \in D$
- $r_j(t)$: Latent fuel demand flow at period $t$
- $x_{i,j}(t)$: Fuel flow from oil terminal $i$ to municipality $j$
- $\beta$: Disappearance rate of fuel demand.
Fuel Assignment Model (intra-temporal model)

Basic Model:

\[
\begin{align*}
\min_{x(t), X(t)} & \sum_{i \in O} \sum_{j \in D} c_{i,j} x_{i,j}(t) \quad \text{(min. total transp. cost)} \\
\text{s.t.} & \sum_{j \in D} x_{i,j}(t) \Delta t = p_i(t), \quad \forall i \in O, \quad \text{(flow reservation)} \\
& \sum_{i \in O} x_{i,j}(t) \Delta t + X_j(t) = q_j(t), \quad \forall j \in D, \quad \text{(flow reservation)} \\
& x_{i,j}(t) \geq 0, \quad \forall (i, j) \in O \times D, \quad \text{(flow non-negativity)} \\
& X_j(t) \geq 0, \quad j \in D \quad \text{(stock non-negativity)}
\end{align*}
\]

- \( c_{ij} \) : transportation cost from \( i \) to \( j \)
- \( p_i(t) \) : fuel production
- \( q_j(t) \) : revealed demand
Model Expansion to Take Fairness into Account

- Measurement of inequality: a weighted entropy of "unresolved rate" of revealed demand

\[ \mathcal{H}(X) := - \sum_{j \in D} q_j(t) \left( \frac{X_j(t)}{q_j(t)} \right) \ln \left( \frac{X_j(t)}{q_j(t)} \right) = - \sum_{j \in D} X_j(t) \ln \left( \frac{X_j(t)}{q_j(t)} \right) \]

  - \(X_j(t)\): unresolved demand at \(t\) (carried over to \(t + 1\))
  - \(q_j(t)\): revealed demand at \(t\)

- Fuel assignment model with fairness consideration

\[
\min_{x(t),X(t)} \sum_{i,j} c_{i,j} x_{i,j}(t) - \theta \mathcal{H}(X) \quad \text{s.t. (1) } \sim (4)
\]

  - \(\theta\): a weight parameter of the inequality against the total transportation cost
Parameter Estimation I

- Relevant parameters
  Fuel distribution model is characterized by two parameters:
  - $\beta$: disappearance rate of fuel demand
  - $\theta$: weight of the inequality against the transportation cost

- Total sales in each prefecture
  Denote the total sales in prefecture $k$ under $(\beta, \theta)$ for a certain duration $\bar{T}$ by

$$\hat{S}_k(\beta, \theta) = \sum_{t \in \bar{T}} \sum_{j \in D_k} \sum_{i \in O} x_{i,j}(t) \Delta t = \sum_{j \in D_k} s_j(t) \Delta t n$$

- $s_j(t) := \sum_{i \in O} x_{i,j}(t)$: fuel sales flow in $j$ at period $t$;
- $D_k$: set of municipalities in prefecture $k$. 
Best estimator
Find the best parameter pair \((\beta^*, \theta^*)\) that explains the actual sales

\[
(\beta^*, \theta^*) := \arg\min_{\beta, \theta} \sum_{k \in K} |S_k - \hat{S}_k(\beta, \theta)|
\]

- \(K\) : set of prefecture;
- \(S_k\) : actual fuel sales in prefecture \(k\) for the duration \(\tilde{T}\).
Data

Model settings

| set of period, $T$ | 03/12 ($t = 1$) to 04/14 ($T = 35$), 2011 |
| oil terminal set, $O$ | Aomori, Hachinohe, Sakata, Sendai-Shiogama and Morioka |
| municipality set, $D$ | 165 municipalities in Tohoku area |
| target duration, $\bar{T}$ | 03/01 to 03/31, 2011 |

Available data set

<table>
<thead>
<tr>
<th>data name</th>
<th>duration</th>
<th>precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) the total gasoline sales March to April, 2010</td>
<td>monthly, by prefecture</td>
<td></td>
</tr>
<tr>
<td>(b) the total gasoline sales March to April, 2011</td>
<td>monthly, by prefecture</td>
<td></td>
</tr>
<tr>
<td>(c) the amount of gasoline transported into oil terminals 03/12 to 04/14, 2011</td>
<td>daily, by oil terminal</td>
<td></td>
</tr>
<tr>
<td>(d) the population of municipality 2010</td>
<td>by municipality</td>
<td></td>
</tr>
</tbody>
</table>

Model inputs

<table>
<thead>
<tr>
<th>model input</th>
<th>data source</th>
</tr>
</thead>
<tbody>
<tr>
<td>oil production ${p_i}$</td>
<td>(c)</td>
</tr>
<tr>
<td>latent demand rate ${r_j}$</td>
<td>(a),(d)</td>
</tr>
<tr>
<td>actual sales $S_k$</td>
<td>(b)</td>
</tr>
</tbody>
</table>
The best parameter is \((\beta^*, \theta^*) = (0.130, 44.0)\), under which the fuel sales of each prefecture is estimated as follows:

<table>
<thead>
<tr>
<th>prefecture</th>
<th>actual sales ((S_k))</th>
<th>estimated sales ((\hat{S}_k))</th>
<th>relative error ((\epsilon_k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aomori</td>
<td>80,666</td>
<td>80,676</td>
<td>0.01%</td>
</tr>
<tr>
<td>Iwate</td>
<td>47,994</td>
<td>49,145</td>
<td>2.40%</td>
</tr>
<tr>
<td>Miyagi</td>
<td>62,877</td>
<td>63,215</td>
<td>0.54%</td>
</tr>
<tr>
<td>Akita</td>
<td>64,758</td>
<td>69,701</td>
<td>7.63%</td>
</tr>
<tr>
<td>Yamagata</td>
<td>39,074</td>
<td>32,636</td>
<td>-16.48%</td>
</tr>
</tbody>
</table>

\[ \text{total error} (\epsilon) = \sum_k \frac{|\hat{S}_k - S_k|}{S_k} \]

relative error: \(\epsilon_k := \frac{\hat{S}_k - S_k}{S_k}\), total error: \(\epsilon := \frac{\sum_k |\hat{S}_k - S_k|}{\sum_k S_k}\)
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Spatial Distribution of Demand-Supply Gap

Demand-supply (D-S) gap of gasoline in municipality $j$ at $\tau$:

$$ G_j(\tau) := \frac{\sum_{t=1}^{\tau} s_j(t) \Delta t}{\sum_{t=1}^{\tau} \{r_j(t) - \beta X_j(t - 1)\} \Delta t} = \frac{\text{Cmltv supply by } t}{\text{Cmltv realized demand by } t} $$

- $s_j(t) = \sum_{i \in O} x_{i,j}(t)$: fuel sales flow in $j$ at $t$;
- $r_j(t)$: latent fuel demand flow in $j$ at $t$;
- $\beta X_j(t - 1)$: disappeared fuel demand during the end of $t - 1$ to the beginning of $t$. 
The subsequent 3 slides show:

1. It was hard to resolve the D-S gap in the areas along the Pacific Sea. Especially, even 3 weeks after the earthquake, the D-S gaps remained in **Miyagi** and **Iwate**, whereas those in other prefectures were almost resolved;

2. At each day, the D-S gap was greater in the **eastern-southern** regions compared to the **western-northern** regions.
Spatial Distribution of Demand-Supply Gap (1st week)

- Aomori (3.15)
- Hachinohe (3.25)
- Akita (3.14)
- Sakata (3.14)
- Morioka (3.18)
- Sendai-Shiogama (3.21)

March 15 ($t = 4$)

March 18 ($t = 7$)
Spatial Distribution of Demand-Supply Gap (2nd wk)

- 0%
- 0+ ~ 40%
- 40+ ~ 60%
- 60+ ~ 80%
- 80+ ~ 98%
- 98+ ~ 100%

March 22 ($t = 11$)

- Aomori (3.15)
- Hachinohe (3.25)
- Akita (3.14)
- Sakata (3.14)
- Morioka (3.18)
- Sendai-Shiogama (3.21)

March 25 ($t = 14$)
Spatial Distribution of Demand-Supply Gap (3rd week)

March 29 ($t = 18$)

April 1 ($t = 21$)

Aomori(3.15)
Hachinohe(3.25)
Akita(3.14)
Sakata(3.14)
Morioka(3.18)
Sendai-Shiogama(3.21)
Conclusion

- We developed a model for estimating the spatio-temporal distribution of demand-supply (D-S) gap, taking into account the following aspects:
  1. Demand disappearance due to a huge backlog
  2. Fairness of the gasoline assignments between municipalities.

- Model parameters were estimated from available data
- Obtained spatio-temporal distributions of D-S gap, which show
  1. Even 3 weeks after the earthquake, the D-S gaps remained in Miyagi and Iwate, whereas those in other prefectures were almost resolved;
  2. At each day, the D-S gap was greater in the eastern-southern regions compared to the western-northern regions.
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Cumulative Demand and Supply (Definitions)

- \( R_j(\tau) = \sum_{t=1}^{\tau} r_j(t)\Delta t \) : Cumulative latent demand by \( \tau \)
- \( E_j(\tau) = \sum_{t=1}^{\tau} \beta X_j(t - 1)\Delta t \) : Cumulative disappeared (pent-up) demand by \( \tau \)
- \( Q_j(\tau) = R_j(\tau) - E_j(\tau) \) : Cumulative realized demand
Cumulative Demand and Supply

- **Aomori**
  - a: Sendai-Shiogama port is opened
  - b: Hachinohe port is opened

- **Akita**
  - a: Sendai-Shiogama port is opened
  - b: Hachinohe port is opened

- **Yamagata**
  - a: Sendai-Shiogama port is opened
  - b: Hachinohe port is opened

- **Iwate**
  - a: Sendai-Shiogama port is opened
  - b: Hachinohe port is opened

- **Miyagi**
  - a: Sendai-Shiogama port is opened
  - b: Hachinohe port is opened